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Transfer function noise modelling of groundwater level fluctuation using threshold rainfall-based binary-weighted parameter estimation approach

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ABSTRACT

Considerable uncertainty occurs in the parameter estimates of traditional rainfall–water level transfer function noise (TFN) models, especially with the models built using monthly time step datasets. This is due to the equal weights assigned for rainfall occurring during both water level rise and water level drop events while estimating the TFN model parameters using the least square technique. As an alternative to this approach, a threshold rainfall-based binary-weighted least square method was adopted to estimate the TFN model parameters. The efficacy of this binary-weighted approach in estimating the TFN model parameters was tested on 26 observation wells distributed across the Adyar River basin in Southern India. Model performance indices such as mean absolute error and coefficient of determination values showed that the proposed binary-weighted approach of fitting independent threshold-based TFN models for water level rise and water level drop scenarios considerably improves the model accuracy over other traditional TFN models.

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1 Introduction

Prediction of groundwater levels with reasonable accuracy is essential for sustainable groundwater resource management. This is especially critical in arid and semi-arid regions where the groundwater resource is highly utilized for various needs, such as in agriculture, industry and municipal sectors. Time series models are often used for prediction of groundwater levels based on historical data. A time series model is an empirical model for stochastically simulating and forecasting the behaviour of uncertain hydrological systems (Kim *et al.* 2005). These are often useful to model any hydrological system where there is limited availability of data, such as groundwater level fluctuation modelling and forecasting.

In general, stochastic time series models can be classified into two kinds; univariate and multivariate models. Univariate models consist of single variable series that can be modelled by differencing based on the autoregressive integrated moving average (ARIMA) group of models or traditional decomposition-based time series models. On the other hand, multivariate models such as the transfer function noise (TFN) modelling technique involve two or more input variables and their dynamic relationships with the output. For example, rainfall series can be related to water level series and their dynamic relationship can be modelled by the TFN approach. TFN models in which the input time series are linearly transformed to output series have been used extensively in modelling groundwater levels by various authors (Knotters and Van Walsum 1997, Van Geera and Zuur 1997, Bierkens *et al.* 1999, Knotters and Bierkens 2000, Yi and Lee 2004).

Groundwater level fluctuations are highly dynamic in nature. The relationship between rainfall and groundwater level fluctuations is highly nonlinear due to complex physical processes, such as infiltration, percolation and evapotranspiration, governing the movement of water through the vadose zone before reaching the water table. This is further complicated due to groundwater abstraction by pumping. Although there are numerous mechanistic models available to model groundwater level dynamics, they all require a huge amount of data and thorough understanding of the physical processes. However, stochastic-based TFN models translate the nonlinear equation into a set of linearly related equations between input and output variables and do not require a thorough understanding/description of the physical processes or parameters of the system involved. Therefore, TFN models are preferred over mechanistic models when there is paucity in the availability of data, due to the simplicity in estimating model parameters and modelling the output response (Hipel and McLeod 1994).

As rainfall is a major factor affecting the groundwater level response, TFN models are often constructed based on rainfall (input variable) as the variable influencing water level data (output variable) as a response variable. Significant cross-correlation between rainfall and water level data has revealed that there is a strong transfer function relationship between rainfall and water level (Yi and Lee 2004). However, TFN models have also been constructed based on rainfall surplus

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(e.g. rainfall minus potential evapotranspiration) and water level time series data (Tankersley *et al.* 1993, Gehrels *et al.* 1994, Knotters and Van Walsum 1997, Van Geera and Zuur 1997). Since the rainfall and water level relationship is non-linear due to vadose zone physical processes, TFN models can also be constructed based on infiltration rate, which is simulated through the vadose zone and respective water level time series data to improve the accuracy of the model results. Infiltration rate time series data could be obtained by the vadose zone water flow simulation codes such as SWATRE (Knotters and Van Walsum 1997) and HYDRUS (Yi and Lee 2004). However, physical parameters such as soil water retention parameters needed to run SWATRE and HYDRUS models are often not available for many parts of the globe. Therefore, TFN models are primarily constructed with rainfall or rainfall excess as the main input variable to model groundwater level fluctuations.

TFN models can be constructed in four major steps, similarly to any other time series models: (a) model identification, (b) parameter estimation, (c) diagnostic checking and (d) model application (Box and Jenkins 1976). Model identification is generally done by specifying the model orders of a TFN model, which has two major components: a dynamic component and a noise component. The dynamic component is usually modelled using rainfall as a variable, which explains part of the water level fluctuation, and the remaining unexplained part of the water level variable is modelled as the noise component. Instead of independently modelling the dynamic and the noise parts of the TFN models, some authors have modelled both components by relating them in terms of their model orders. With such conditions, TFN models can be represented as autoregressive exogenous variable (ARX) models and autoregressive exogenous moving average variable (ARMAX) models (Knotters and Bierkens 2000). ARX models are constructed on only the dynamic part of TFN models, where the noise component is assumed to be normally distributed with independent residuals. ARMAX models are developed on both dynamic components and the correlated noise components. Parameter estimation for ARX and ARMAX models is generally done by least square minimization and prediction error techniques, respectively. Diagnostic checking is done for the calibrated time series models in which the modelled errors are verified for normality and non-dependency (no serial correlation). After the diagnostic checking of the residuals, selected models can be used for prediction and forecasting applications.

A major advantage of TFN models is that they can be combined with physical models and compared with physical parameters. The physical basis of the rainfall–water level relationship can be compared with simple ARX model parameters (Knotters and Bierkens 2000). Physically-based rainfall–groundwater level models are then used to predict the intervention effect of rainfall quantity over the dynamics of groundwater level. A simple model order of the TFN model structure, such as ARX and ARMAX models, is most often considered for simplicity in estimating the time series model parameters. This also facilitates comparison of the TFN model parameters with the physical model parameters such as drainage resistance, infiltration rate and regional groundwater flux.

A special set of TFN (ARX) model parameters can be developed using both water level rise and water level drop events together (implicitly) along with the corresponding rainfall data. This does not introduce appreciable error on a daily time scale; however, when the time series model is built on a monthly time scale, large errors may be introduced due to uncertainty in the estimation of TFN model parameters as opposed to accounting for rise and drop events explicitly. In most parts of the world, either monthly or bi-monthly groundwater level is routinely observed, as opposed to daily measurement. In this study, a novel approach is adopted to estimate threshold-based ARX model parameters by splitting the rainfall–water level data, based on a rainfall threshold, into water level rise and water level drop series identified from the calibration dataset.

2 Literature review

Time series models have been used extensively in groundwater hydrological applications (Salas *et al.* 1982, Adamowski and Hamory 1983, Houston 1983, Furbish 1991, Lee and Lee 2000).Viswanathan (1983) modelled the rainfall and water table relationship for a coastal unconfined aquifer using a first-order autoregressive time series model (FOARX) at Tomago sand beds near Newcastle, New South Wales, Australia. The model structure was formed with the water level at day t as dependent on rainfall from day t to t-8. The FOARX model parameters were estimated using a recursive algorithm by minimizing the cost function between the predicted and observed water level values. The rainfall and water level data were used in the parameter estimation stage without considering water level rise or drop events. The FOARX model parameters, λ, α and β, were associated with the water level variable, rainfall variable, and external disturbances, respectively. These statistical parameters were correlated to interpret the physical parameters of drainage factor, infiltration factor and other factors influencing rainfall–water table depth in a hydrological process, respectively.

In a similar study, Kim *et al.* (2005) designed time series models to evaluate groundwater discharge characteristics for subway systems in Seoul, South Korea. Time series of rainfall and groundwater discharge observed at three subway stations, Gireum, Garibong and Sadang, were used to develop TFN models using monthly rainfall and groundwater discharge data. The autocorrelation function (ACF) and cross-correlation function (CCF) plots were used to identify a proper TFN model structure for every station of the subway system in Korea. Three different combinations of TFN models were identified based on ACF and CCF results for each subway station. Akaike information criterion (AIC) and Bayesian information criterion (BIC) values were computed for all identified sets of TFN models at every subway station. The best TFN model was selected based on the minimum values of AIC and BIC model indices. The selected TFN models were used to predict the stream discharge against rainfall data for corresponding subway stations. Model validation results showed that good agreement was achieved between the model predicted discharge rates and observed discharge rates. Although their study was based on monthly data, they did not assign differential weights for monthly rainfall during water level rise and water level drop periods, which might further reduce the uncertainty in the model predictions.

Changnon *et al.* (1988) developed a statistical relationship between monthly rainfall and groundwater levels for 20 wells scattered across Illinois, USA. The monthly differenced water levels and rainfall series were modelled using TFN models. An individual autoregressive integrated moving average (ARIMA) model was fitted for rainfall and water level data separately until the stationarity condition was satisfied. Pre-whitened rainfall and water levels were correlated using cross-correlation analysis and a lag of 1 month was observed between rainfall and groundwater levels. Three different forms of transfer function models were compared using the AIC index and the best one was recommended for further modelling of rainfall–water level relationships. Estimated transfer function model parameters were correlated with the geomorphological conditions of the site to understand the underlying physical relationships based on the response of groundwater to rainfall, and thus the information was used for drought assessment of the region. The parameters of the transfer function model at various sites were combined, from which a single equation with respect to soil parent material was developed. The developed transfer function model for precipitation and shallow groundwater levels was used to predict water levels during a drought at any location in Illinois, excluding 20 sites for which water level information was available. Model predictions at each site were compared with actual monthly water level data over 20 years. The predicted values were within one standard deviation of measured water levels except for two values. Average differences of 27 cm to 47 cm during the cold season months and 31 cm to 48 cm during the warm season showed that the derived equation could be applied for areal estimates of shallow groundwater levels during the drought events. In this study also, monthly rainfall during water level rise and water level drop events was treated similarly when fitting the TFN model parameters.

Knotters and Van Walsum (1997) developed a set of physically-based time series models (SWATRE-TFN) to estimate daily fluctuations of water table depths with rainfall excess as the daily input variable. The SWATRE model accounts for the nonlinear physical relationship between the groundwater head and rainfall variables by simulating unsaturated groundwater flow. The unexplained part of the SWATRE model output was modelled by a noise component with ARMA model parameters based on Box and Jenkins time series methodology (Box and Jenkins 1976). Validated results of SWATRE combined with TFN noise model estimates performed better than using standalone TFN models.

Yi and Lee (2004) developed a TFN model with groundwater head as output series and rainfall and infiltration rate derived from the HYDRUS package as input series to the model. This TFN model was built on regularly observed (daily) input–output data series, using a Kalman filtering technique to integrate with irregularly observed water level series to estimate groundwater heads at the unobserved time. The CCF between groundwater heads and rainfall may be misconstrued as an exact lag relationship between input and output data series due to the inherently high autocorrelation tendency of groundwater levels. Pre-whitening-based cross-correlation analysis was suggested in this study to identify an exact cross-correlation relationship between rainfall and groundwater level. The model orders (r, s, p, q, b) during the calibration period were determined as (1, 1, 1, 1, 0) for one well. For another well located in the same formation, they were identified as (1, 2, 1, 1, 0). The results of the selected TFN models were validated with 1000 d water level data. Mean error, mean absolute error (MAE) and root mean square error (RMSE) model indices showed that the TFN model predictions were satisfactory.

Most of the time, TFN models are constructed with a linear relationship between rainfall or rainfall excess and water level. But, from physical evidence, the relationship between rainfall and water level is known to be highly nonlinear. As the saturation of the root zone process is highly time dependent in the unsaturated zone, the corresponding evapotranspiration, percolation and recharge processes varies with time. Therefore, it might lead to large uncertainty if this process is avoided while constructing TFN models for groundwater level fluctuation predictions. To address this problem, Berendrecht *et al.* (2006) modelled the unsaturated zone processes using Richards' equation and Darcy's law to estimate the time series of evapotranspiration, percolation out of the root zone and recharge. A nonlinear state-space model was developed with an extended Kalman filter algorithm to calibrate its parameters with the observed water level series. For comparison purpose, a linear TFN model was also calibrated with the observed water level series. Results showed that the nonlinear model performed better than the linear model and improved the prediction accuracy of the groundwater levels.

Another type of nonlinearity in rainfall–water level fluctuation modelling arises due to drainage flux occurring at different drainage levels. Knotters and De Gooijer (1999) developed nonlinear state space models with shifting water level regimes, commonly known as threshold autoregressive self-exiting open-loop (TARSO), to model groundwater levels with different levels of drainage separated by thresholds. TARSO model results were better than the linear TFN and dynamic regression (DR) model results as the model incorporated different regimes resulting from different soil layers and drainage levels.

In another similar study, the nonlinearity due to drainage in groundwater systems was addressed by Berendrecht *et al.* (2004). They developed a state-space threshold model to account for the nonlinearity, which was constructed with measured groundwater table depth along with precipitation and evapotranspiration time series. A maximum likelihood criterion was adopted to estimate the parameters, which included the threshold value for the drainage level. Application of this model was tested on two time series of water level datasets and the results showed the superior performance of the state-space model in predicting groundwater levels. This model was also used in characterizing the groundwater systems, as the physical basis in terms of drainage levels was incorporated in the model rather than just the water level prediction itself.

The groundwater system is influenced by both natural climatic conditions and human interventions. Linear TFN models often ignore the nonlinearity arising from human interventions, such as groundwater withdrawals, when modelling groundwater level fluctuations. Gehrels *et al.* (1994) developed a linear stochastic TFN model for groundwater table depth prediction from rainfall excess time series data and then separated the artificial component from the natural groundwater regime. Trends in groundwater time series data arise due to natural and artificial causes. TFN models are a useful tool to decompose such processes and model them independently, as well as modelling them as a whole. Van Geer and Defize (1987) decomposed groundwater in terms of natural and artificial components using TFN models. The effect of the artificial component of groundwater pumping was assessed by the developed TFN models.

Simple and special cases of TFN models, such as ARX models with few parameters, have been extensively developed based on the physical processes of the vadose zone rainfall–water level relationship to predict water level fluctuations (Knotters and Bierkens 2000, Bierkens *et al.* 2001). Although very often used to capture the linear relationship between rainfall and water level response, the estimated parameters of the ARX models are still uncertain due to the mixed effects of water level rise and water level drop events inherent in the actual water level data. This is especially true in the case of modelling with monthly time series, where uncertainty in estimation of ARX model parameters is high, as opposed to accounting for rainfall during water level rise and water level drop events explicitly. Therefore, a threshold rainfall-based binary-weighted least square method was adopted to parameterize the TFN models in this study to account for the nonlinearity in the rainfall–water level fluctuation process.

The objective of this study is to model groundwater level by comparing ARX model performance resulting from two different ways of parameterizing the ARX models using the least square technique as follows:

(1) The traditional method of estimating linear ARX model parameters using the continuous rainfall–water level calibration dataset.
(2) Considering threshold rainfall and explicitly identifying water level rise–rainfall and water level drop–rainfall regimes in estimating ARX model parameters using a binary-weighted approach.

In order to assess the efficacy of the ARX models, a generic univariate time series model, deseasonalized ARMA (Ds-ARMA), for water level data was developed as the base model with which the ARX models are compared.

3 Methodology

3.1 TFN-based ARX models

TFN models consist of two components, dynamic and noise. The dynamic component of a TFN model involves one or more input variables that explain part of the variability observed in the output signal. The unexplained part of a TFN model is independently modelled by a noise process. TFN models can be constructed based on standard time series methods with the proper steps necessary to model the stochastic time series process. Single input–single output transfer function models for rainfall and groundwater level data were given by Knotters and Bierkens (2000) as:

$$h_t = h_t^* + N_t \tag{1}$$

$$h_t^* = \sum_{i=1}^{p} a_i h^*{}_{t-i} + \sum_{j=0}^{q} b_j p_{t-j-k} \tag{2}$$

$$N_t - \mu = \sum_{i=1}^{r} c_i (N_{t-i} - \mu) + \sum_{j=1}^{s} d_j \varepsilon_{t-j} + \varepsilon_t \tag{3}$$

where h_t is the water table depth at time t [L]; h_t* is the water table depth attributed to the rainfall value [L]; N_t is the unexplained part or noise term [L]; p_t is the average rainfall attributed to the time step $t-1$ to t [L]; k is the delay factor between input and output responses; μ is the expected value of the noise term [L]; a_i is the autoregressive parameter of the transfer function model of order $i = 1, \ldots, p$; b_j is the moving average parameter of the transfer model of order $j = 1, \ldots, q$; c_i is the autoregressive parameter of the noise model of order $i = 1, \ldots, r$; d_j is the moving average parameter of the noise model of order $j = 1, \ldots, s$; ε_t is the white noise with mean zero and variance σ^2.

The delay factor (k), given in Equation (2), is identified by the cross-correlation of the pre-whitened white noise series of rainfall and water level data. The TFN model orders such as p, q, r, s were kept as simple model orders for ease of comparison among different models with different parameterization methods. Therefore, in this study we selected a TFN model with model orders, $p = 1$; $q = 0$; $r = 1$; $s = 0$; $k = 0$. By applying the selected model orders in Equations (1)–(3), the resulting TFN model is as follows:

$$h_t = h_t^* + N_t \tag{4}$$

$$h_t^* = a_1 (h^*{}_{t-1}) + b_0 (p_t) \tag{5}$$

$$N_t - \mu = c_1 (N_{t-1} - \mu) + \varepsilon_t \tag{6}$$

If the model order of the autoregressive parameter of the noise model is taken to be same as the autoregressive parameter of the transfer model, i.e. $c_1 = a_1$, then the TFN model of Equations (4)–(6) can be reduced to a special case of the TFN model, which is also known as the ARX or ARX(1,0) model, as follows:

$$h_t - \mu = a_1 (h_{t-1} - \mu) + b_0 (p_t) + \varepsilon_t \tag{7}$$

3.1.1 ARX model parameter estimation using a least square technique

The traditional method of estimating parameters of an ARX model using a simple least square technique accounts totally for the input variables, such as past water level data and present rainfall data, during the calibration period. Consider the transformed form of Equation (7):

$$H_t = a_1(H_{t-1}) + b_0(P_t) + \varepsilon_t \tag{8}$$

where $H_t = h_t - \mu_h$ is the transformed groundwater level [L]; $P_t = p_t$ is the rainfall series [L] (Equation (7)); μ_h is the minimum water level [L].

Traditional least square ARX (TLS-ARX) model parameters were estimated by the following steps:

Step 1. For the given input series of groundwater levels $[H(t-1), H(t-2), \ldots]$ and rainfall $[P(t), P(t-1), \ldots]$ the information matrix and parameter vector can be formulated as:

$$\begin{aligned}\hat{H}(t) &= a_1 H(t-1) + b_0 P(t) \\ &= [H(t-1)|W \times P(t)] \times \begin{bmatrix} a_1 \\ b_0 \end{bmatrix} \\ &= \Phi(t)^T \theta, \quad W = 1\end{aligned} \tag{9}$$

Step 2. Prediction error ε at time t can be calculated as:

$$\varepsilon(t,\theta) = H(t) - \hat{H}(t) = H(t) - \Phi(t)^T\theta \tag{10}$$

Step 3. Sum squared error can be minimized to estimate the parameter θ given by:

$$\mathrm{Min}(\theta) = \sum_{i=1}^{N} \varepsilon(t,\theta) \times \varepsilon(t,\theta)^T \tag{11}$$

Step 4. The matrix form of Equation (11) for time $t = 1, \ldots, N$ that minimizes the error (E) and estimates the respective parameters is:

$$\mathrm{Min}(\theta) = E(N,\theta) \times E(N,\theta)^T \tag{12}$$

$$\hat{\theta}_{\mathrm{TLS}} = [\Phi(N)^T\Phi(N)]^{-1}\Phi(N)^T H(N) \tag{13}$$

Step 5. The model prediction equation during the validation stage for the TLS-ARX model is:

$$\hat{H}_t = \hat{a}_{\mathrm{TLS}}(H_{t-1}) + \hat{b}_{\mathrm{TLS}} P_t \tag{14}$$

3.1.2 ARX model parameter estimation using a binary-weighted least square technique by considering water level rise and drop explicitly

Unlike traditional least square parameter estimation methods, in binary-weighted least square schemes, two independent ARX models with unique parameterization are developed for (1) data pairs of rainfall and water level during the water level rise events with rainfall above a threshold and (2) data pairs of rainfall and water level during the water level drop events.

The steps involved in this binary-weighted water level rise and drop ARX (BW-RD-ARX) model parameter estimation were as follows:

Step 1. First a threshold rainfall value (T_P) above which water level rise was encountered was identified. This was done by sorting the data based on the water level rise and identifying the corresponding minimum rainfall value.

Step 2. In the model calibration stage, water level rise or drop events were identified based on observed water level data at current and previous time steps ($H(t)$ and $H(t-1)$). A subset of the rainfall–water level dataset was identified in which rainfall values corresponding only to water level rise periods and rainfall above the threshold value (T_p) were given full weighting, while rainfall during water level drop periods was suppressed by imposing a weight of zero. Similarly, another model was built with the rest of the data corresponding to water level drop events, by suppressing rainfall corresponding to water level rise by imposing a weight of zero. This is further explained in Equations (15) and (16) as follows:

$$\begin{aligned}\hat{H}(t) &= [H(t-1)|W \times P(t)] \times \begin{bmatrix} a_{\mathrm{BW-R}} \\ b_{\mathrm{BW-R}} \end{bmatrix} \\ &= \Phi(t)^T\theta_{\mathrm{BW-R}} \begin{cases} W = 1 & \text{if } H(t) - H(t-1) > 0, \text{ and } P(t) > T_p \\ W = 0 & \text{otherwise} \end{cases}\end{aligned} \tag{15}$$

$$\begin{aligned}\hat{H}(t) &= [H(t-1)|W \times P(t)] \times \begin{bmatrix} a_{\mathrm{BW-D}} \\ b_{\mathrm{BW-D}} \end{bmatrix} \\ &= \Phi(t)^T\theta_{\mathrm{BW-D}}, \begin{cases} W = 1 & \text{if } H(t) - H(t-1) < 0 \\ W = 0 & \text{otherwise} \end{cases}\end{aligned} \tag{16}$$

The parameters of Equations (15) and (16) are estimated by the usual method of minimizing the sum square error using Equations (10)–(13).

Step 3. As the current time step water level is not known during the model validation or forecasting stage, water level rise and water level drop events were identified based on the previous two time steps in observed water level data ($H(t-1)$ and $H(t-2)$). It was assumed that water level rise or drop for the current time step is most likely dependent on the previous two consecutive water level values. Therefore, the model prediction equation during the validation stage for the BW-RD-ARX model is given in Equations (17) and (18) as follows:

$$\hat{H}_t = \hat{a}_{\mathrm{BW-R}}(H_{t-1}) + \hat{b}_{\mathrm{BW-R}}(P_t), \quad H_{t-1} - H_{t-2} > 0, \text{ and } P_t > T_p \tag{17}$$

$$\hat{H}_t = \hat{a}_{\mathrm{BW-D}}(H_{t-1}) + \hat{b}_{\mathrm{BW-D}}(P_t), \quad H_{t-1} - H_{t-2} < 0 \tag{18}$$

It should be noted that the predictions for BW-RD-ARX model parameters $\hat{a}_{\mathrm{BW-R}}$, $\hat{b}_{\mathrm{BW-R}}$, $\hat{a}_{\mathrm{BW-D}}$ and $\hat{b}_{\mathrm{BW-D}}$ from Equations (17) and (18) are totally different from the TLS-ARX model parameters $\hat{a}_{\mathrm{TLS}}$ and $\hat{b}_{\mathrm{TLS}}$ from Equation (14), as they are estimated only on the basis of a subset of the calibration dataset, whereas TLS-ARX model parameters are estimated from the complete calibration dataset without any differential weighting.

3.1.3 Deseasonalized ARMA model

A univariate time series model, Ds-ARMA, for water level data was developed as a base model with which the TFN-based ARX models described in Sections 3.1.1 and 3.1.2 were compared. Ds-ARMA models are basically developed on transformed time series data where long-term monthly averages are estimated from the original time series data and the seasonality is removed from the original data. As Ds-ARMA models are a well-known and effective univariate modelling technique for streamflows and groundwater level with significant seasonality in nature, such a model was considered in this study to compare it with the proposed ARX models (Govindasamy 1991, Peng and Liu 2000, Almedeij and Al-Ruwaih 2006, Mondal and Wasimi 2006, Modarres 2007, Fernandez *et al.* 2008, Paul 2008, Ghanbarpour *et al.* 2010, Martins *et al.* 2011, Adhikary *et al.* 2012, Lu *et al.* 2014). A deseasonalized series was modelled with simple AR and MA components. The Ds-ARMA approach according to Hipel and McLeod (1994) is given by:

$$\phi(B)(h_t - \hat{\mu}_{mt}) = \theta(B)e_t \tag{19}$$

with coefficients:

$$\phi(B) = 1 - \phi_1 B - \ldots \phi_p B^p$$
$$\theta(B) = 1 - \theta_1 B - \ldots \theta_q B^q$$

where h_t is the observed water level data [L]; $\hat{\mu}_{mt}$ is the mean monthly value for month m at time t [L]; $\phi_1 \ldots \phi_p$ are the autoregressive coefficients of order p; $\theta_1 \ldots \theta_q$ are the moving average coefficients of order q; B is the backshift operator; e_t is the white noise [L].

For simple case comparison, the AR and MA terms of the Ds-ARMA model were modelled with one coefficient in each (i.e. ϕ_1 and θ_1, respectively). Parameters of the Ds-ARMA model were estimated by a maximum likelihood technique. The prediction equation of the Ds-ARMA model is as follows:

$$\hat{h}_t = \hat{\mu}_{mt} + \phi_1(h_{t-1} - \hat{\mu}_{mt}) - \theta_1(e_{t-1}) \tag{20}$$

3.2 Model assessment indices

The model efficiency was assessed using model indices such as MAE, RMSE and R^2. The formulations for MAE (E_{MA}), RMSE (E_{RMS}) and R^2 are:

$$E_{\mathrm{MA}} = \frac{1}{N}\sum_{t=1}^{N}|H_t - \hat{H}_t| \tag{21}$$

$$E_{\mathrm{RMS}} = \sqrt{\frac{1}{N}\sum_{t=1}^{N}(H_t - \hat{H}_t)^2} \tag{22}$$

$$R^2 = 1 - \left(\frac{\sum_{t=1}^{N}(H_t - \hat{H}_t)}{\sum_{t=1}^{N}(H_t - \bar{H})}\right)^2 \tag{23}$$

where N is the sample size; H_t is the observed value at time t [L]; $\hat{H}_t$ is the predicted value at time t [L]; $\bar{H}$ is the observed mean value [L].

4 Case study

4.1 Study area

The efficacy of the proposed binary-weighted ARX model (BW-RD-ARX) over traditional ARX models (TLS-ARX) and Ds-ARMA models was tested on groundwater level data from the Adyar basin, India. The Adyar basin is located in the northeast coastal part of the state of Tamil Nadu, India (Fig. 1). This basin receives rainfall from both southwest and northeast monsoons. Northeast rainfall is predominant and occurs during the months of October, November and December. The southwest monsoon rainfall is erratic in nature and summer rainfall is negligible. Long-term annual average rainfall is about 1315 mm (WRO 2007). Climatic conditions in the sub-basin are classified as dry humid and semi-arid tropics. Elevation ranges from 183 m above mean sea level (a.m.s.l.) in the west to sea level in the eastern part of the basin. Soils in the basin have been classified into clayey, black, red sandy and alluvial. Black soils occur in the depressions adjacent to hilly areas in the west. Alluvial soils occur along the river courses and in the eastern part of the coastal areas. The major hydrogeology in the basin is classified as unconsolidated, semi-consolidated and weathered fractured rock formations. Groundwater occurs under phreatic and semi-confined conditions in intergranular pore spaces in sands and sandstones, and in bedding planes and thin fractures in shales. The groundwater table depth fluctuation in the observation wells varies from a minimum near the surface to a maximum at 12 m below ground level.

4.2 Data used

Daily rainfall data for five raingauge stations in and around the Adyar basin were acquired from the State Ground and Surface Water Resources Data Centre (SG&SWRDC), Taramani, for a period of 15 years (1988–2002). Monthly water level data were also acquired for 26 observation wells for the same time period. Data were available for a period of 180 months (1988–2002), out of which 120 months (1988–1997) were used for calibration and remainder was used for validation. Observation well depth to water table data were converted to hydraulic head data with respect to the datum plane of m.s.l. Based on the areal extent of the Thiessen polygon, four rainfall zones were identified and the observation wells were categorized into one of these four zones. The proposed approach was applied to all 26 wells from the study area. However, for brevity, the time series plot for only one well is presented here for illustration. The selected well (Well 13172) is located in the Kovalam rainfall zone at 12.87°N latitude and 80.24°E longitude (Fig. 1).The time series of selected well data along with the rainfall values are shown in Figure 2. The model performance in terms of MAE and R^2 for all 26 wells is presented below in the form of maps for a comprehensive assessment of the proposed approach of building TFN models.

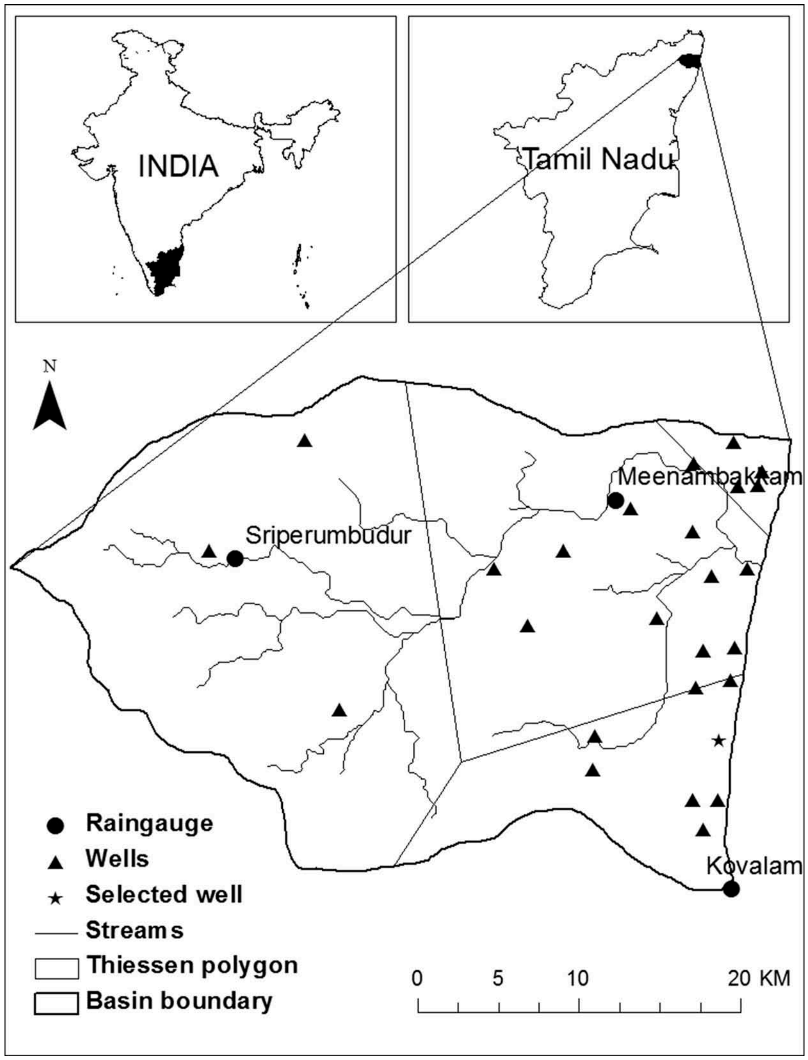

Figure 1. Drainage map of the study area along with observation well and raingauge locations.

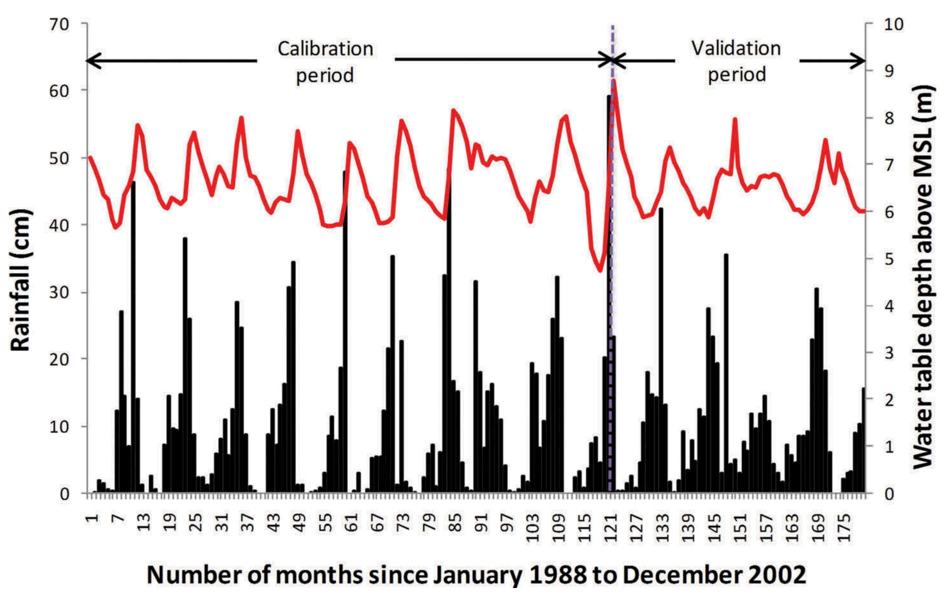

Figure 2. Rainfall–water level data of Well 13172 used in ARX models.

5 Results and discussion

5.1 Autocorrelation and cross-correlation analysis of rainfall–water level data

Autocorrelation and cross-correlation analyses were made on the rainfall, water level and rainfall–water level data for each of the 26 wells to identify the time series model structure. As expected, the ACF plot (Fig. 3) shows that the monthly rainfall and water level data have significant autocorrelation due to seasonality.

Cross-correlation analysis on actual rainfall and water level data shows a sinusoidal pattern, indicative of seasonality (Fig. 4(a)), from which the exact cross-correlation lag relationship between rainfall and water level data cannot properly be revealed. Hence, the rainfall and water level data were both pre-whitened and a cross-correlation analysis was carried out on the pre-whitened residuals.

Pre-whitening is the process of identifying an appropriate ARMA model to convert the residual to a white noise. Individual rainfall and water level time series data were fitted with appropriate ARMA models to identify the cross-correlation between rainfall and water level data. As both rainfall and water level data were affected by the seasonality, they were deseasonalized before identifying suitable ARMA models (deseasonalized ARMA) for both the variables according to Box and Jenkins (1976). ARMA filters for the deseasonalized rainfall and water level data for Well 13172 are given in Equations (24) and (25). Deseasonalized ARMA filter for rainfall data:

$$(1 - 0.05B - 0.52B^2)(p_t - \mu_p) = (1 - 0.36B + 0.64B^2 + 0.35B^3)e_t \tag{24}$$

Deseasonalized ARMA filter for water level data:

$$(1 - 0.64B)(h_t - \mu_h) = (1 - 0.37B)e_t \tag{25}$$

Uncorrelated residuals obtained from the deseasonalized ARMA models for rainfall and water level were compared by cross-correlation analysis. A CCF plot of pre-whitened rainfall and water level data was constructed for up to 20 months lag. The CCF plot clearly shows that the cross-correlation is maximum at lag zero for the given rainfall and water level data (Fig. 4(b)). Furthermore, the lag zero cross-correlation is the only significant correlation between rainfall and water level data. Hence, the lag time parameter was fixed as zero (k = 0) in the ARX model order specification.

5.2 Estimation of model parameters

Taking model parsimony into account, other model orders for the autoregressive transfer function model and moving average transfer function model were fixed with one parameter in each term; therefore the complete model orders for the ARX model were p = 1; q = 0; r = 1; s = 0; k = 0 and the

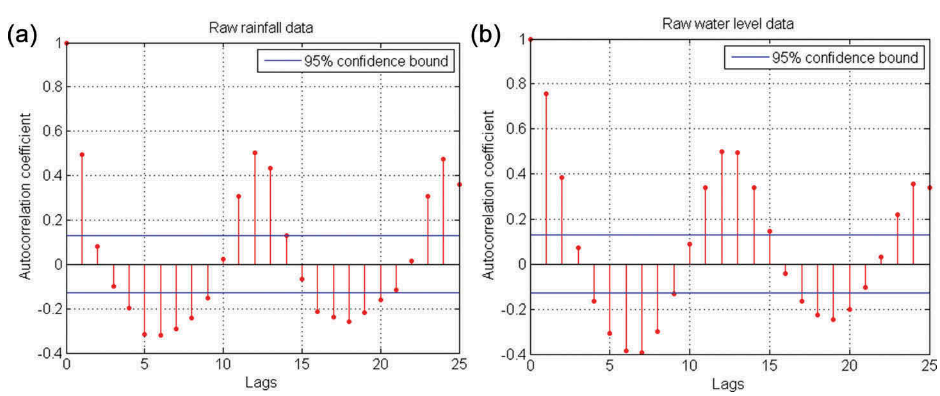

Figure 3. Autocorrelation plot of actual (a) rainfall and (b) water level data of Well 13172.

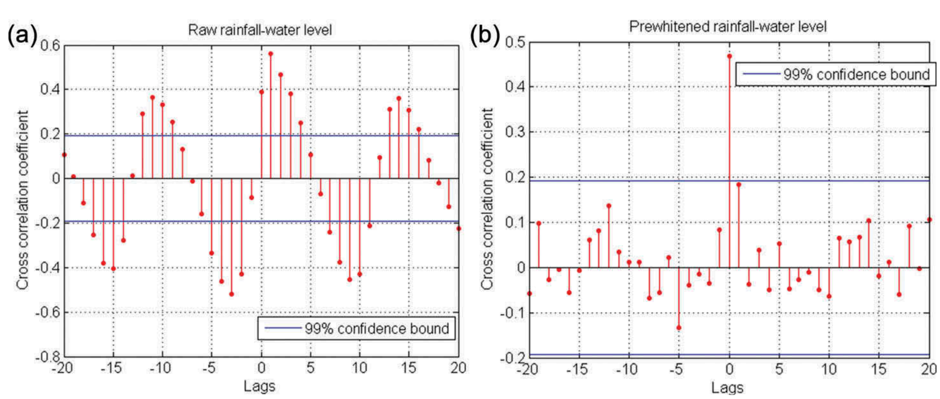

Figure 4. Cross-correlation plot of (a) actual rainfall–water level data and (b) pre-whitened rainfall–water level data of Well 13172.

Table 1. Estimated parameters of Ds-ARMA and ARX models for Well 13172.

Model	Parameters	Value	Standard error	*t*-statistic	*p*-value
Ds-ARMA	ϕ_1	0.61	0.06	9.52	0.00
	θ_1	0.63	0.08	8.36	0.00
TLS-ARX	a_{TLS}	0.80	0.03	25.81	0.00
	b_{TLS}	0.03	0.00	12.42	0.00
BW-RD-ARX	$a_{BW\text{-}R}$	0.94	0.02	40.69	0.00
	$b_{BW\text{-}R}$	0.01	0.00	4.19	0.00
	$a_{BW\text{-}D}$	0.94	0.03	32.91	0.00
	$b_{BW\text{-}D}$	0.02	0.01	2.73	0.00

corresponding model is given in Equation (7). The parameters of Equation (7) were estimated by both the traditional least square error method and the binary-weighted least square error method. The TLS-ARX model estimates the model parameters based on the entire water level and rainfall data without accounting for water level rise and water level drop events separately, whereas the BW-RD-ARX method estimates model parameters by explicitly identifying water level rise and water level drop events from the calibration dataset.

In addition to these ARX models, a univariate time series model, similar to the one given in Equation (20) for water level data was also separately developed using a deseasonalized approach for comparison with other ARX models. The Ds-ARMA and ARX models were fitted using 10 years of monthly data for 1988–1997. The Ds-ARMA model and the ARX model parameters with their corresponding statistics are given in Table 1.

Note that all the estimated model parameters were significantly different from zero, as their *p*-values were less than 0.01 (99% confidence level) (Table 1). The residuals of the univariate (Ds-ARMA) and bivariate models (ARX models) were analysed for residual independence and normality conditions. Residual independence was checked by plotting ACF plots of the simulated model residuals and it was observed that the serial correlations for lag of up to 20 months were insignificant (within 99% confidence interval), which indicates that there was no evidence for serial correlation. Therefore, the developed models showed a high degree of confidence in terms of their predictions and they could be compared in terms of their performance.

The same procedure of model fitting and residual tests was carried out at other locations where water level and rainfall data were available and similar results were obtained (no serial correlation). Therefore, the developed models were applied independently at all the locations for predicting groundwater levels.

5.3 Model validation

One-month rolling predictions of water table depth were made using the developed ARX and Ds-ARMA models and validated using observed data for the period of 5 years from 1998 to 2002. Based on the model structure identified using ACF and CCF analysis, in the case of Ds-ARMA models, previous month deseasonalized water level data (autoregressive term) and previous month correlated error data (moving average term) were used to predict the current month water levels. In the case of TLS-ARX models, current month rainfall data along with the past month observed water level data were used for predicting the current month water level value. In the case of the BW-RD-ARX model, when the data for water level at the past two time steps and corresponding hydraulic head change are positive and the corresponding rainfall value is higher than the threshold rainfall value, the BW-RD-ARX model predicts the water level rise mode using the corresponding estimated ARX model parameters. When either one of the above conditions is not satisfied, then the BW-RD-ARX model predicts the water level drop mode using the corresponding estimated ARX model parameters.

Prediction results show that the BW-RD-ARX model performs significantly better than the other traditional univariate (Ds-ARMA) and bivariate (TLS-ARX) models (Fig. 5(c)). The accuracy of the BW-RD-ARX model performance in terms of MAE, RMSE and R^2 values for the selected well (13172) was significantly better than the other two models (Ds-ARMA and TLS-ARX), as shown in Figure 5. Overall linear dependency of the developed models was evaluated in terms of R^2 values. The high R^2 value for BW-RD-ARX indicates that the model reliability in predicting the water levels with rainfall values is high when compared with the other two models. Model performance in terms of deviation was determined by MAE and RMSE model indices. MAE and RMSE values for the BW-RD-ARX model were significantly lower those for the Ds-ARMA and TLS-ARX models, which indicates that this model predicts the water levels with less deviation from the observed values.

5.4 Effect of range of rainfall magnitude on water table depth predictions

The accuracy of the developed ARX models (TLS-ARX and BW-RD-ARX) was further analysed for different magnitudes of monthly rainfall values. Validation datasets containing monthly rainfall values, observed water table depths and predicted water table depths for the developed models were binned based on the monthly rainfall thresholds in ascending order, from which three different ranges of monthly rainfall values (0–10 cm, 10–20 cm and >20 cm) and corresponding observed and predicted water levels were taken independently. Observed and predicted water levels from the TLS-ARX and BW-RD-ARX models for these three threshold limits of monthly rainfall were plotted on a 1:1 line (Figs 6–8).

It was observed that the predictions by the BW-RD-ARX for all ranges of rainfall values were better than those by the TLS-ARX model for corresponding rainfall ranges (Figs 6–8). Interestingly, the BW-RD-ARX model predicts with the highest accuracy for the rainfall range of 0–10 cm as compared to other rainfall ranges of 10–20 cm and >20 cm (Fig. 6(b)) over the TLS-ARX model. This is probably because at higher ranges of rainfall the runoff process may become dominant due to saturation of the vadose zone and any further increase in rainfall would not produce an equivalent increase in the recharge process.

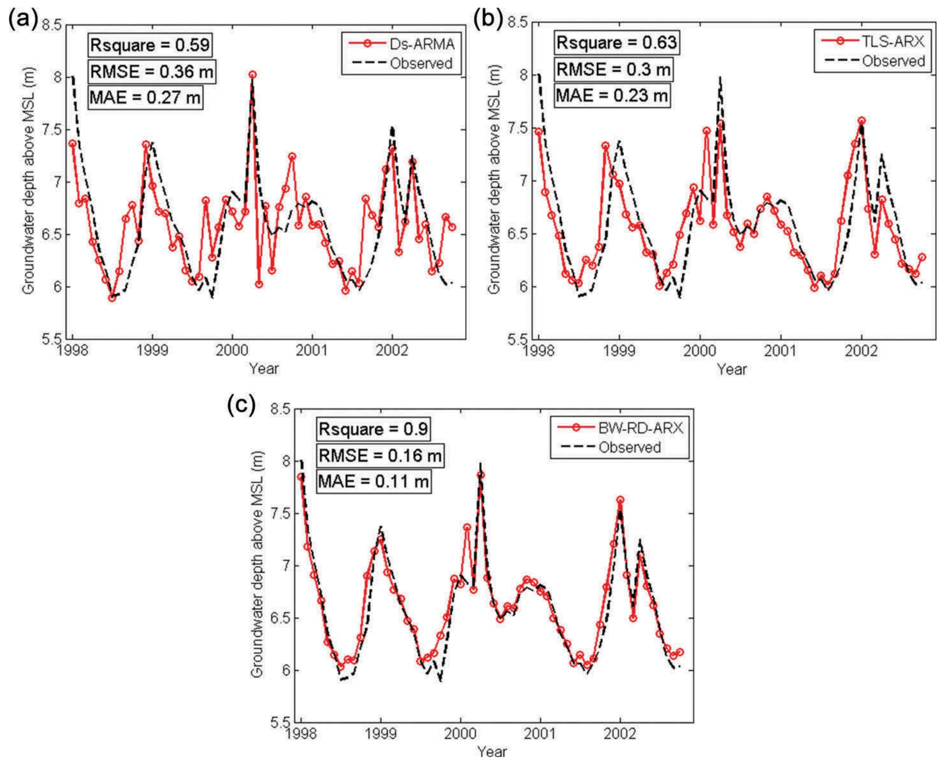

Figure 5. Water levels predicted by (a) Ds-ARMA, (b) TLS-ARX and (c) BW-RD-ARX models.

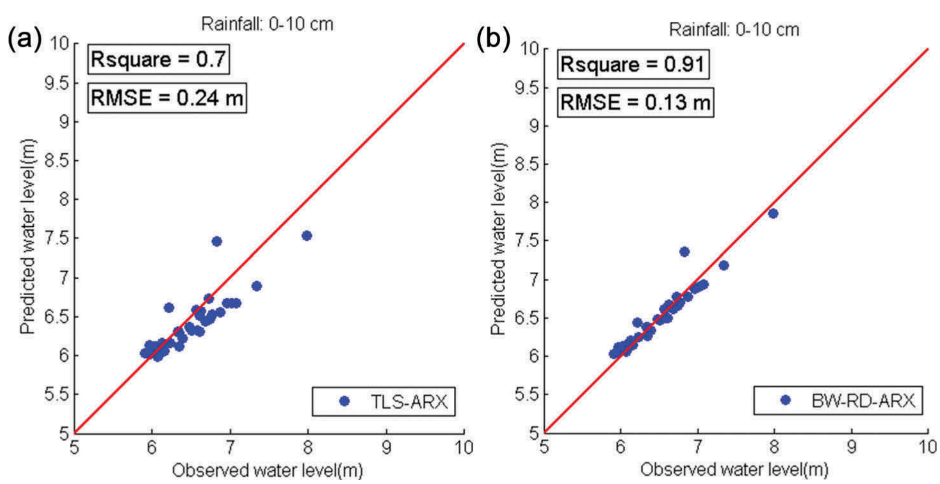

Figure 6. Observed and predicted water levels for rainfall less than 10 cm by (a) TLS-ARX and (b) BW-RD-ARX models.

Similar observations on model performance were made for all 26 wells. Overall, the BW-RD-ARX model outperformed other models at all ranges of rainfall magnitude. Therefore, application of the BW-RD-ARX model in predicting groundwater levels is highly valid over the wide range of rainfall distribution regions that occur in semi-arid to humid climatic conditions.

The prediction performance for the BW-RD-ARX model was better than that of the TLS-ARX model because uncertainties in the AR and transfer model coefficients of the BW-RD-ARX model were greatly reduced, as they were logically estimated with two independent models by separating water level–rainfall data pairs into water level rise and drop events. Considerable uncertainty remains in the estimated AR and transfer model coefficients of the TLS-ARX model because they were estimated with a water level–rainfall dataset in which water level data implicitly combined water level rise and water level drop events.

5.5 Spatial interpolation of model indices over the study region

The developed univariate and transfer function models were evaluated for all 26 observation wells in the study region. Model performance measures such as MAE and R^2 were calculated during the validation period (1998–2002) from the respective time series models. The ordinary kriging approach was adopted for spatial interpolation of MAE and

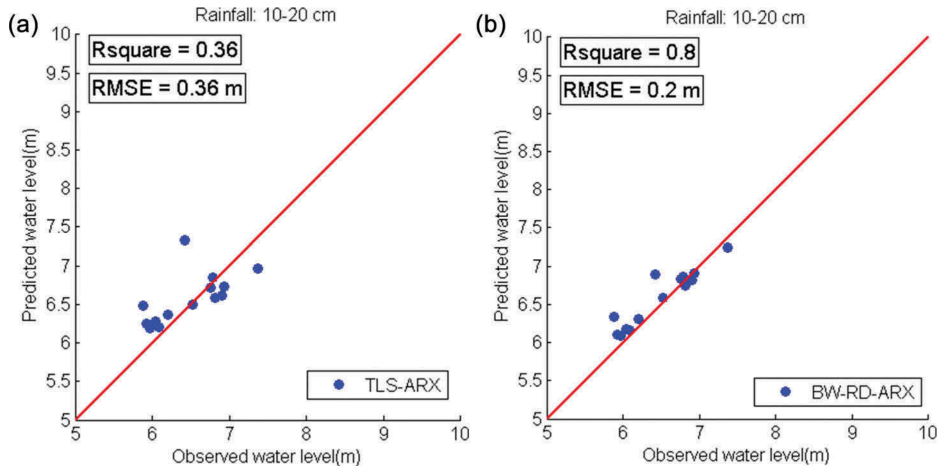

Figure 7. Observed and predicted water levels for the rainfall range 10–20 cm by (a) TLS-ARX and (b) BW-RD-ARX models.

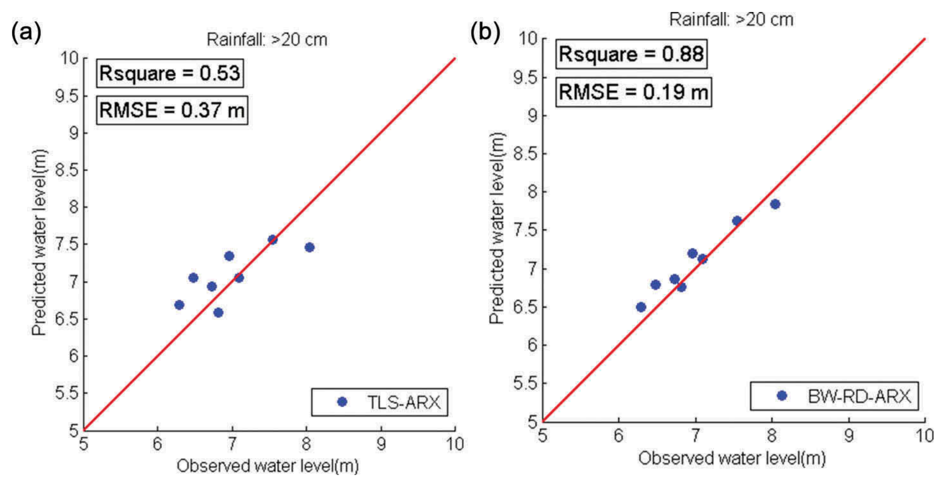

Figure 8. Observed and predicted water levels for rainfall greater than 20 cm by (a) TLS-ARX and (b) BW-RD-ARX models.

R^2 over the study region. A spherical semi-variogram was fitted before interpolating the values using the kriging approach. Spatial interpolations of the MAE values from the developed models are given in Figure 9.

MAE values varied highly from 0.02 m to 1.3 m over the study region. The places where the higher gradient MAE values were observed are in regions with high altitude. Variations in the observed water level data at these places were high, due to mountains where the presence of rock fractures could have caused the water level to vary significantly at any time during the observation period. All the developed models comparably predict water levels with high MAE values in the central part of the study region (Fig. 9). However, the BW-RD-ARX model had the lowest MAE value (0–0.5 m).

In the eastern coastal part of the study region, where the density of observation wells is higher, all the models consistently predicted water levels with lower MAE (0–0.6 m). In the western part of the study region, where the observation wells are fewer in number, MAE values were observed in the range of 0.2–0.9 m. A major part of the western region had low MAE values (0.2–0.5 m) predicted by BW-RD-ARX when compared to other models (Fig. 9).

Spatial interpolation of R^2 values was done by the same kriging approach and keeping the same semi-variogram type (spherical) for all the models (Fig. 10). As far as R^2 variation over the study region is concerned, BW-RD-ARX performed best across the entire watershed, with R^2 values above 0.91. The Ds-ARMA model performed worst, with R^2 values less than 0.7 across most of the domain.

6 Summary and conclusions

Understanding the behaviour of the groundwater system for different climatic stresses such as rainfall is important for proper planning and management of groundwater resources. Rainfall–groundwater level relationships can be effectively modelled by TFN-based ARX models. But the traditional ways of estimating linear ARX model parameters have considerable uncertainties as the rainfall–water level relationship becomes highly nonlinear. Therefore, in this study, a binary-weighted method of estimating the ARX model parameters was adopted under two scenarios. Two independent ARX models were developed based on two different datasets,

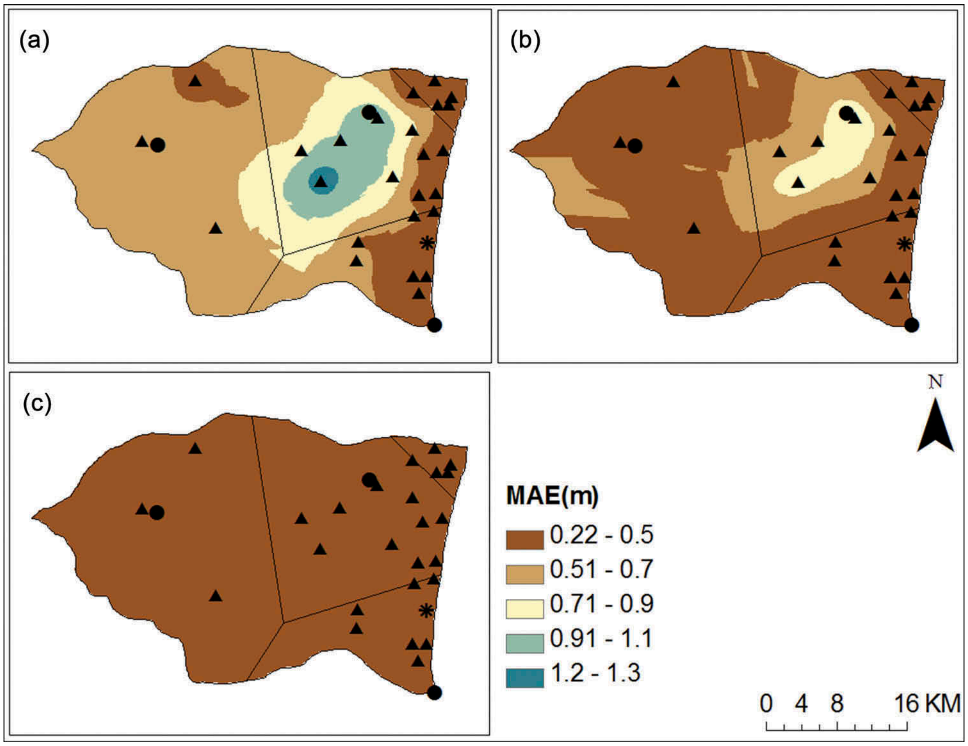

Figure 9. Spatially interpolated MAE index from (a) Ds-ARMA, (b) TLS-ARX and (c) BW-RD-ARX models.

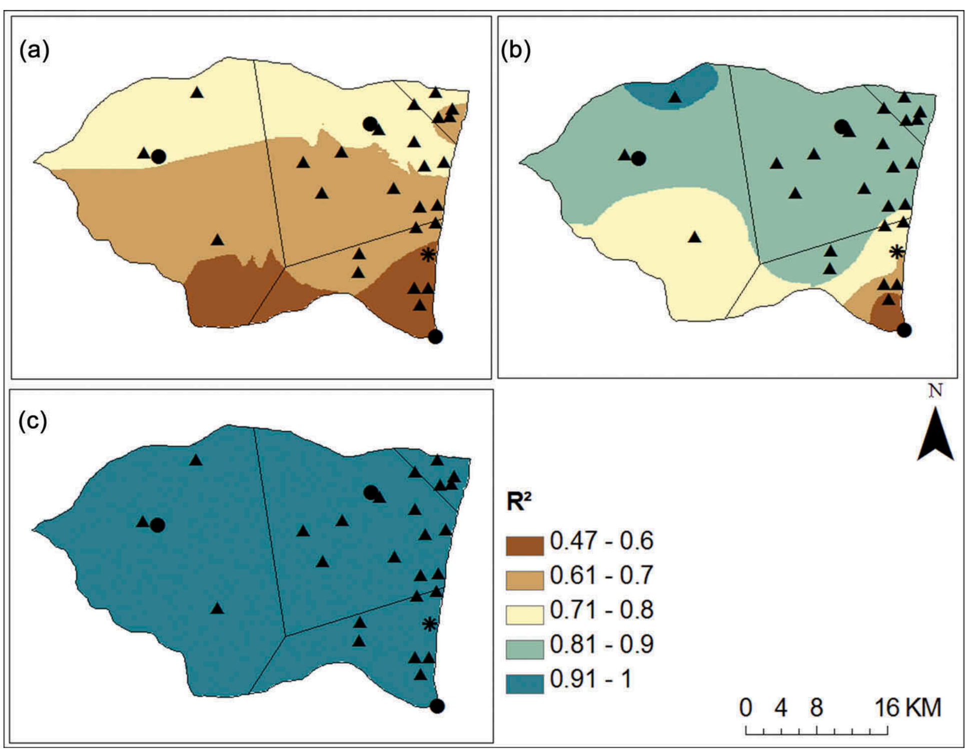

Figure 10. Spatially interpolated R^2 index from (a) Ds-ARMA, (b) TLS-ARX and (c) BW-RD-ARX models.

which were identified based on water level rise and drop events.

It was observed from the TLS-ARX model results that it could not perform effectively in predicting water table depths at various ranges of rainfall, which in turn could lead to poor decisions in managing the groundwater resources. However, the BW-RD-ARX model performed better in predicting water levels for all ranges of rainfall (0–10, 10–20 and >20 cm) and therefore prediction results could lead to better decision making in water resources management.

The results at discrete well locations in the study zone showed that the binary-weighted method of estimating the TFN model (BW-RD-ARX) was more effective in predicting groundwater levels when compared to traditional ways of estimating the TFN model and univariate time series model. Spatially interpolated MAE and R^2 values among all the developed models showed that the BW-RD-ARX model performed significantly better than the other models (Figs 9 and 10). In general, it was observed from the interpolated R^2 and MAE values that Ds-ARMA and TLS-ARX model predictions were comparatively poor at high-altitude regions. However, the BW-RD-ARX model predictions were promising in such places. In spite of the good performance of the BW-RD-ARX model, there is further scope for improving the modelling framework and hence the accuracy of predictions.

- In this study, rainfall and water level data were modelled using a TFN-based ARX approach as rainfall and water level data are widely available for many places. However, the nonlinear relationship between rainfall and water level process could not be captured by the ARX models as it was developed based on linear relationships between input and output data. However, instead of rainfall, percolation or recharge time series generated from the physically-based flow models can be used in an ARX modelling approach, which might further improve the model results.
- In this study, a binary-weighted least square approach was tested for water level rise and drop events of water level data irrespective of the specific seasonal water level rise and drop events. If the binary weights were further subdivided according to the monsoon/non-monsoon seasonal water level rise and drop events, the resulting optimized parameters of the ARX model might further improve the model efficiency.
- A point-based temporal modelling of water table depths may give further insights into groundwater response for an external input such as rainfall only at a particular location, which cannot be interpreted precisely over a large scale (basin scale). This can be effectively analysed by a spatio-temporal modelling of water level depths based on the regional ARX models by a physically-based ARX with ancillary data such as DEM, soil data and other relevant hydrogeological data.

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Disclosure statement

No potential conflict of interest was reported by the authors.

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